Memorias del Seminario en Conmemoración de los 400 Años del Nacimiento de René Descartes. Santafé de Bogotá: 1997, págs. 51-92. ISBN 958-9205-26-7

ANALYTIC GEOMETRY, EXPERIMENTAL TRUTH AND METAPHYSICS IN DESCARTES

Mario Laserna

Carrera 3, no. 9-52, Santafé de Bogotá, D. C., Colombia

Introductory remarks. [(1)] Our aim is to reconstruct the way Mind, according to Cartesius, attains true and certain knowledge of nature's laws. That reconstruction involves two different processes: A historical one depicting how through *applying* the mathematical-experimental method objective knowledge of reality *has* been attained. A second, epistemological one, presents Cartesius' analysis of the mathematical-experimental method as the *cause* of objective cognition as an *effect*. It explains how knowledge of natural phenomena *necessarily* results from applying a cognitive strategy deployed by Mind. A method of discovery has been invented!

To prove the above contention we construct two scenarios *H* and *H*; a historical one, *H*; and a personal one *D* built on the basis of his quest for rational, certain truth, as told by Cartesius. In *H* (the historical one) the meta-language of physics is presented *a posteriori* as capable of providing an object-language in which the truth of judgements referring to space-time events has been, historically, attained. *Judgements true only potentially may be entertained*; a sub-method (experimental testing) is provided for *systematically selecting only true ones*. In scenario *D* the Cartesian proposal for a meta-language in which the possibility, on the part of Mind, of formulating functional-relations (the object-language), *i.e. possible* ways in which nature is constituted, is presented. A discovery method for mapping object-language entities upon space-time events (experiment) selecting from *among the set of functional relations (intellectus)* those *adequate* to the real world, is rationally justified.

The difference between *H* and *D* is the following: *H* presents the meta-language of physics *warranted, a posteriori, by four centuries of applying the mathematical-experimental* method. From Galileo, and Newton down to Hertz, Planck, Einstein, de Broglie, Yukawa, quarks and leptons. Scenario *D reserves* the time sequence holding in *H* between object-and meta-language; *D* shows, *a priori* of *historical praxis, the possibility (principles and concepts) of the process presented in H, were formulated by Cartesius*. How? By endowing Mind with the powers to act as cause of the *effect scientia*, *i.e.* of *true and certain knowledge*. The meta-language generating the object-language of *scientia* establishes the mental operations required to provide rational coherence in the object-language. Such a reversal of serial order entails not only the abandonment of Aristotelian realism, but also that the object-language

of *scientia is not a natural but an artificial language*; a construct of reason in which grammar (syntax) prescribes, following Mind's cognitive strategy, rules for its use. It also implies that the historical objective Mind, through the action of some preestablished harmony, expresses thoughts identical with those issuing from the mind of a certain René Descartes whose personal mind appeared on 31 March 1595 at La Haye, disappearing in Stockholm on the 11 of February, 1650 because of a pneumonia.

Our essay, consequently -on the basis of well-known texts-, has as main purpose *demonstrating* that the historical Mind and the personal mind of Cartesius in relation to the meta-language of *scientia* are equivalent, constitute and epistemic unity. Our problem, consequently, is *not* whether the individual Cartesius directly or indirectly influenced historical argumentations or *dramatis personae appearing* in *H*; nor that what Cartesius stated or misused in the object-language is or ever was valid; nor whether his mistakes in his use of the object-language are related to his invention of a valid meta-language of *scientia*. Our contention is restricted to the following: The information provided by his story warrants asserting that he attained an insight valid even for present-day micro- and macrophysics into the relationships holding between the object- and the meta-language of Physics. In other words, *Cartesius, invented the correct and unerring rational criteria applied today for evaluating scientia.*

Finally, and before considering in detail the announced scenarios, the key to establishing the epistemic equivalence of *H* and *D* lies in the invention / discovery of Analytic Geometry, and the epistemic weight Cartesius attributes to that discovery regarding the formal nature of mathematics. Consequently, this essay is divided in three Sections. Section I presents the conceptual structure of scenario *H*. Section II, constructs scenario *D* on the basis of the story told in *The Discourse on the method for correctly conducting reason and finding the truth in the Sciences*. (The title itself heralds its Meta-method *nature and intention!*) Finally, Section III briefly presents three Cartesian themes which, in my opinion, play a central role in present-day philosophical debate, *ad portas* of the 3rd Millennium.

SECTION I

Serial order, elements and truth in the process of knowledge

1.1. From the particular to the general, a two-way epistemic process. Sensorial daily-life perceptions can never be all-embracing for each act, of its very nature, spans only a limited area of possible experience; *sense-knowledge is of particulars*. By relaying impressions to the central agency called 'brain', *the senses merely furnish the raw material which, duly processed, produce an information -proposition transformable into a judgement from which in its turn originates a behavior command.*

The act of judging being the fundamental *cognitive* act of our mental faculty, proffering a judgement turns out to be a highly complex act of synthesis integrating epistemic-logical elements of a filo- and onto-genetic nature. An act of judgement

represents, *from an epistemic point of view, the construction of a truth-value within a linguistic-cognitive scenario*; a last act in a succession, emerging from those syntactic entities called *'propositions'*. Consequently, a reality-oriented judgement can be seen as resulting from a syntactic form (provided by Mind not by sense perception) *which, placed in the conveyor-belt of an individual's cognitive activity undergoes a routine process of being filled-in with constants furnished by sense; acquiring, in its last stage, truth-value status*. The whole, perceived by the subject as an spontaneous *psycho-biological* process, constitutes the normal way of relating to reality through our innate linguistic faculty! Hence, *the task assigned to Philosophy by Cartesius:*

i) to reconstruct and evaluate the conditions (mental, and organic) under which such a process takes place;

ii) to determine its constitutive elements and,

iii. to evaluate the truth status of the final product. In the use of speech, a philogenetic activity of normal human beings, its most important functions and categories are *experienced in toto*, not piece-meal. Only analysis reveals that a gradual filling-in process, *transforming an individual experience into an intersubjective truth-value*, has taken place.

The Cartesian contention is to the effect that by following true principles certainty of truth can be attained, both as to the world we live in *as well as to the truthfulness of the method and serial order deployed in the cognition process itself*. With regard to the cognition process it challenges the Aristotelian realism of the schools in the following points:

1. The final product, *viz.* a cognition judgement, may alter the quality of its truth-value if the serial-order undergone during the filling-in process is changed. The conveyor-belt permits different serial orders for the filling-in of what is, initially, a pure syntactic form.

2. For the cognitive conveyor belt the commutative law of Arithmetic to the effect that $a + b = b + a$ does not hold among the elements constitutive of a truth-value.

3. The *entire process*, mouthing in a truth-value, *whatever the serial order adopted*, is organized and steered by Mind according to its own cognitive strategy.

4. A linguistic-scenario (Mathematical, or Table of Judgements one) constitutes a prerequisite for a truth-value to emerge.

5. The epistemic origin of the syntactic form (natural language *vs.* mathematical constructs) plays a *decisive* role in the historic confrontation between Cartesian and Peripatetic cognitive systems.

The importance of a rationalist, anti-positivist, albeit space-time reality oriented epistemology issues from being an *alternative to the specific conveyor-belt serial order of cognition imposed by naive realism*: An alternative providing an scenario in which mathematical-experimental physics can be rationally construed; the truth of its results being warranted by Mind itself. The historical confrontation between sensual-realism and experimental-demonstrative rationalism does not originate in the nature of the cognitive end-product which is, in both cases, the reality reported

by common-sense speech; moreover, the final product contains, in both (realism and rationalism), the same elements of sense and understanding, *albeit integrated through a different serial order* specific to the language employed. The Aristotelian-realism Cartesian mathematical-empiricism confrontation originates in the degree of certainty accruing to the end-product; and originates in the serial order in which the elements are synthetized during the cognitive process. We become conscious of the existence of a filling-in process by following the common-sense method used by Gottlob Frege for reaching some of his paramount discoveries: Through analyzing some paradigmatic filling-in operations connected with elementary mathematics and logic [(2)].

The filling-in process carried out during the cognition process takes place, *e.g.* when the function i) expressed in arithmetical speech:

i) ;·· + ··· = ···, becomes the false truth-value.

j) 7+5 = 13, by filling-in with the integers, 7, 5 and 13.

Through the filling-in process i), a pure syntactic form receiving an arithmetical interpretation becomes, j). Clearly pure form i) filled-in with numerals representing numbers antecedes any act of arithmetical calculation [(3)]. It also shows that the filling-in process within daily-life language can be carried out in a different order, *i.e.* instead of going from the particular to the general, it is possible, reversing the serial order followed by naive realism, to go from the general to the particular. The sequence-order between a concept and its extension can as in the case of geometrical concepts, be reversed. For such a reversal the question immediately arises: Whence does the concept originate if not by abstracting from perceived individuals? Can acts of judgement take place if the individual is not previously given as really existent? The answer of geometry since Thales has been: Mind thinks space-time relations $F(x,y)$ under the guise of mathematical functions subsisting between potential, not actual entities. Thereupon, in order to get an experimental truth-value we look around in space-time events to see if a pair of individuals j an k satisfy $F(x,y)$, *i.e.* if $F(x,y)$expresses a relation experimentally holding between j and k, That we do, for example, with Galileo's spheres rolling down a plane; or during a sun eclipse, observing the curvature of light rays in the vicinity of great masses. We then say that spheres rolling down an inclined plane constitute an empirical model for $F(x,y)$; or that Einstein's Relativity theory correctly predicts the behavior of light rays in the vicinity of great masses of matter.

If that is *scientia*, what is the purpose of our daily-life description of the world, so intimately united with our senses? Is God, after all, cheating us? A dilemma is clear: either God is a deceiver or we are mistaken in believing in daily-life experience is the gate to *scientia*. Faced with the dilemma, Cartesius reinterprets *nihil est in intellectus quod prius non fuerit in sensu*, rejecting its first horn. Hence, in the sixth *Meditation on First Philosophy*, the last half of which is dedicated to establishing the survival value of sensorial experience: *My own nature teaches me ... that when I feel pain there is something wrong with my body ... and so to seek out what induces feelings of pleasure, ... and from the fact that I perceive many varieties of colours, sounds, smells ... it does not seem to follow that its purpose is to teach us about things*

located outside of us ... The traditional *primum vivere deinde philosophare* explains the variety of our mind's powers.

Since the Table of Judgements provides an in-built relation between individual and concept, realism teaches that the concept *horse* arises from our experience of individual horses. Our mental faculties, however, do not come to a still-stand at the level of the experience of particular sensorial events. Our linguistic scenario *consists of particular and general statements in which particular and general terms interact with one another*. On the basis of such *scenarios we inevitably conclude that daily-life language faithfully mirrors the epistemic structures of naive realism*. Experience, however, extends beyond what can be expressed in the logic of the schools. Different manners of concept formation are required in order to express different epistemic structures. A reality beyond the one constituting our daily-life meso-cosmos demands a language adequate to its own micro an macro structures.

1.2. The historical awareness of the crisis of realism. The last two or three generations of scholarly research have made clear the stages undergone by modern science in its historical development. Already in the 12th century inquisitive individuals aware that the language of geometry describes cognitive events relating the external world significantly differing from those originating in sense-particulars.

The more we deploy logical-linguistic structures and criteria differing from those in-built into naive realism, the more a reversed sequence in the ensuing filling-in is demanded. Critically oriented minds like Robert Grosseteste, Kepler or Francis Bacon, began asking the *how* and the *why*, using the language of mathematics ; motion may be described with a precision impossible to attain within Aristotelian epistemic schemata. *Such inquiries belonging to the meta-language of Science mouthing, in the course of time, into the School of Padua, give rise to the Scientific Revolution of the 18th century*. The unity of knowledge demands that one and the same epistemic language should serve for expressing functional-relations at levels of reality different from the familiar ones of our meso-cosmos. How are such different realities to be discovered and explained?, becomes the important question to which Galilean physics provides an answer through the idea of a theoretical model. It suffices that a certain supposition as to how known events occur also have the power to entail, anticipate verifiable events to become accepted within the rationality supporting the existence of the real world. Such is the meaning of *Thales, ou qui que ce soit, qui a dit le premier que la lune reçoit sa lumière du soleil, n'en a donné sans doute aucune autre preuve, sinon qu'en supposant cela, on explique fort aisément toutes les diverses faces de la lumière: ce qui a été suffisant pour faire que, depuis, cette opinion ait passé par le monde sans contredit*. The same relation between the light of the moon we perceive as an effect, to the truth that it is caused by a reflection of the sun-rays -something we merely assume without any direct perception of it since prior to the Astronauts it could not be perceived by human eyes- applies to the explanations provided for certain operations of Mind present in cognition. What the reference to Thales is supposed to demonstrate *is not that the moon actually reflects the light of the sun*, but that, as a matter of fact (a fact regarding the very meaning of the term explanation) a cause-effect explanation will be accepted as valid when it results from a model establishing a causal common-sense connection

corresponding to the reality of the case. For the text continues *Et la liaison de mes pensées est telle, que j'ose espérer qu'on trouvera mes principes aussi bien prouvés par les conséquences que j'en tire, lorsqu'on les aura assez remarquées pour se les rendre familières, et les considérer toutes ensemble, que l'emprunt que la lune fait de la lumière est prouvé par ses croissances et décroissances* [(4)]. What is worth noticing in this argument is that the criteria for accepting a causal explanation for physical phenomena is applied also to an explanation of non-sensorial phenomena namely, how, when acting on true principles, Mind proceeds to construct a causal model for the act of cognition deserving general acceptance. In this manner the unity of reason is not only preserved; the importance of an adequate grasp of first principles is high-lighted, and the validity of arguing from effect to cause attains the status of a general epistemic category.

However, the necessity of maintaining the unity of rational truth originating in the daily-life *paradigma* is not our problem in the present paper. Calling attention to the selectivity character of sense-information simply should serve as a reminder that an analogous situation presents itself when dealing with a system of philosophy spanning many themes as is the case with the Cartesian one. The possibility of selecting, *culling a specific fruit from the tree of philosophy* is not alien to the Cartesian system. The fruit we intend to cull is of a non-physical nature; it contains the meta-not the object-language of science (to put it in the contemporary speech of Philosophy of Science). Our intentions are to *provide sufficient textual evidence* demonstrating that Descartes explicitly and coherently formulated the basic epistemic elements and operations of Mind required *for the meta-language of mathematical-experimental physics*. In particular he saw the role played by a purely formal mathematics combined with experimental testing for physical theory as practiced in the 20th century. It is also clear that the role of philosophy is to establish the meta-language warranting the object-language of the different sciences, which according to the principles of philosophy are there: Medicine, Mechanics and Morals.

1.3 From daily-life to scientific knowledge. Knowledge of daily-life reality becomes *the paradigma* for all knowledge involving sensorial data. Being an original and spontaneous starting point from our experience of *res extensa* given in daily-life it should be no surprise that when advances in cognition beyond daily-life appear in the cognitive scenario, nonetheless, the same paradigma provides *the* measuring rod and criteria *applicable to more sophisticated forms of cognition appearing in the scenario of history*. The very meaning of truth as an *adequatio* between linguistic utterances and non-linguistic entities appears as in-built into our use daily-life language. To the point that only through Frege's doctrine declaring that the assertion of truth is not a predicate within the proposition but a *propositional attitude* similar to the *interrogative* or the *imperative* one, has the matter reached a promising degree of clarity.[(5)] However, such a philogenetic[(6)] origin of our cognitive paradigma in no way implies that all knowledge must *begin* with sense perception. Nor that there can exist no logical forms other than those of Aristotelian logic for describing space-time events (as Kant erroneously assumed). It does mean, however, that as far as cognition of the world in which human beings find themselves is concerned, howsoever we may construct it, knowledge must *work its way back* to its daily-life paradigma. A principle of unity valid for all reality, warranting thus the

application of pure science to practical affairs which, apart from its theoretical importance, lies at the very root of the conceptual possibility, historical emergence, and posterior globalism of the Industrial Revolution. The use of mathematics in physics does not divorce from, does not destroy, daily-life realities. It signifies a different more precise way of identifying, of referring to them, as stated in the *Entretiens* with Burman (Meditation V*): The difference between mathematics and physics consists merely in the fact that physics looks upon its object not only as a true and real being but also as one in act; and as such existent. Mathematics, on the contrary, considers it only in its possibility; not actually existent in space, yet able to exist.* To establish what is actually existent and not mere possibility is the role of experiment. The true and real being considered a legitimate candidate for special citizenship, must be the object of a *perception claire, non de I'magination*. The very useful explanations as to what constitutes clear perception, not an imagined one is a theme *que I'auteur a passé en revue dans le premier livre des principes* is the final observation to the matter.

The basic principle guiding the search for a scientific interpretation of Nature can be stated in an analogy to the manner the artisans, who through their practical manipulation of specific elements of nature (metals, stone, cloth, time-keeping, domestication of animals, musical theory,...), allow us to partially use Nature for our well-being. Science turns out to be a rational generalization of what artisans accomplish as a result of perfecting, through an age-long practice, their respective trade. Innovations in the Arts are the outcome of mere practice; without guiding rational and systematic principles (*scientia*) warranting a method of discovery. The empiricism of the Arts acquired an epistemic dignity and expression in the logic of the schools and the prevalence of explanations based on tradition and the theory of occult qualities. Magic and occultism, whatever degree of practical control over natural phenomena they promised, were, for Cartesius, no substitute for a rational method of discovery which could be learnt by directing and instructing *common-sense*. A consequence of rationally directing common-sense united to a new quantitative manner for understanding Nature became in-built into the emphasis on method and experimentation. The result was certainty of knowledge, *scientia*. Nonetheless, judging with hindsight one can conclude that limited to the epistemic scenario of naive realism, *i.e.* deprived of principles justifying some novel approach to cognition, it was a more simple task to empirically adopt than to philosophically define truth-criteria of what *scientia* is, in open confrontation with the logic of the Schools, a logic associated to a tradition-bound form of rationality and common sense supportive of empiricism and realism against magic and other criteria defying the peripatetic orthodoxy of the schools[(7)]. A *Zeitgeist* supported by the fact that *Discourse on the Method of rightly conducting the Reason and seeking for the Truth in the Sciences* (henceforth abbreviated to *DM*) begins with the comment: *Of all things in the world common sense is the most equally distributed, for everyone considers himself abundantly provided with it...*[(8)]

Cartesius, on the basis of his visionary experiences of 10.XI. 1619 claimed to have discovered an approach to Nature different from the one taught in the schools[(9)]. Not only was his method inspired in the rationality of mathematics. If required, further proof of its rationality follows from its fruitfulness, compared to the

barrenness of Aristotelianism: *Consequently, when one acts on true principles in matters of philosophy one cannot miss encountering, now and then, other truths; and no better reasons could be given for rejecting the Aristotelian principles than by pointing out that despite following them for centuries no progress has been made,* runs an argument in the Preface to the *Principles of Philosophy*. Voicing, thereby, a complaint, prevalent since the Renaissance, regarding the method of the schools. A complaint presented with considerable coherence and analytic power, by Francis Bacon in *Novum Organum* (lxxiii) to the effect that *Of all signs there is none more certain or more noble than that taken from fruits. For fruits and works are as it were sponsors and sureties for the truth of philosophies*; or regarding the fruitlessness of logic *logical invention does not discover principles and chief axioms, of which Arts are composed, but only such things as appear to be consistent with them*. (lxxxii) The absence of a logic of invention originates in the idols affecting the thinking powers of the Mind plus relying on axioms too abstract to be useful as guides in a logic of discovery. In regard to the Aristotelian appeal to experience in lxiii points out that *... he had come to his conclusions before: he did not, as he should have done ... but having first determined the question according to his will, he then resorts to experience, and bending her into conformity with his placets leads her about like a captive in a procession:* reaching the conclusion *so that even on this account he is more guilty than his modern followers, the schoolmen, who have abandoned experience altogether.* Consequently, to change such a situation a new formulation of the problem of knowledge, including inventing new middle axioms (the Cartesian principles) becomes a dire necessity! *By their fruits you can know them* turns out to be a universal principle, voiced against the logic of the schools !

It was the manifest philosophical destiny of Descartes, having found *the* starting-point leading to cognitive certainty, to discover (hit-upon) the principles leading to a condition of nature endowed with certainty (*scientia*). Thus permitting man to become master -in contrast to the particular ones of the artisans- *of any and every force found in Nature*. It is this momentous discovery of the new principles and the true method for acquiring knowledge of Natural phenomena *which prompted publishing the Discourse on the Method of rightly conducting the Reason and seeking for the Truth in the Sciences*. The reasons presented in *DM6* itself leading to an abandonment of his adopted practice *not to publish* leave no doubt as to his own belief regarding the earth-shaking importance of his *scientia*; nor, if proved to be right, as to the new era it heralded for mankind in its dealings with Nature. His principles leading to *scientia*, have no longer mere speculative use. Hence the decision to make them public.[(10)]

1.4. Nature and the language of mathematics. The great discovery, we are told, presents itself at the level of the method to be applied for attaining *practical* control of physical events, not at that of the speculative sciences: *I have never laid great value on those things originating in my own thinking. And, as long as I culled no different fruits from the Method I use beyond satisfying myself regarding certain difficulties in the domain of the speculative sciences,... I had never considered it an obligation to put them in writing...*(*DM6*). Such a personal consideration implies that the decision to publish *DM* arises from having gained insight into an entirely new circumstance related to a revelation , during his dream of 10. XI. 1619, of the

existence of a Universal Mathematics: As soon as I *had acquired some general notions concerning physics, and having begun to make use of them in relation to various particular difficulties, I became aware to what results they could lead to, and how much they differ from the principles of which we have made use up to the present time, ... I concluded I could not keep them as my personal privilege without incurring in a great sin against the law which commands us to contribute, as a much as possible, to the general welfare of humanity.*

For through applying those principles it is possible to attain knowledge of great use in our life[11]. Was this belief of having discovered the key to systematically unlock the secrets of nature a mere illusion? Or was it based on some rational principle that permits dominating through our human mind the forces constituting nature? The latter seems to have been the case for he concludes: *Through these principles we way find a practical philosophy by means of which, having knowledge of the force and the action of fire, water, air, the stars, the heavens and all other bodies that surround us, as distinctly as we know the trades of our artisans we may apply them to all the uses for which they are appropriate thus becoming masters of nature.*

The practical philosophy statement proclaims physical science (mechanics) as the hand-maiden to technology. For given any natural force we can invent a way of transforming it so that it becomes an instrument for obtaining a benefit from (sailing; wind-mills; water pumps; optical instruments...) nature's processes, with more precision than is done in a restricted number of cases by *the different crafts of our artisans*. However, at this point notice should be taken that the ensuing three treatises following *DM ... the little which I myself have discovered*, namely Dioptrics, on Meteors and the Geometry, do not reveal the specific method employed but simply apply it[12]. The explanation for such an omission being that he wanted to make some of these advances himself, *I have no hesitations in asserting that I shall have no necessity of attaining more than two or three new victories equivalent in kind* (the three treatises) *in order to fulfill my plans. Moreover, my age is not so advanced ...yet I believe to be so much more committed to employ the time which I still have available with the greatest efficiency in the utmost hope of employing it well. I am greatly afraid, however, that I would be robbed of my time were I to make public the foundations of my physics ... since it is impossible they would be accepted. For confronted to the prevailing opinions of diverse groups, I anticipate that I would be frequently distracted from my main design due to the opposition they would awake.* The principles which prompted him to believe he had discovered the method through which the subsequent advances of Physical Science became possible were already part and parcel of what the School of Padua, through Galileo and his disciples, were applying when combining theoretical constructs with experimental validation. It is not, therefore, after a glance at the physics of the 17th century, our contention that without Descartes the advance of Physics would not have taken place. Nor that in *DM* he tells us all there is to know as to the true method[13]. *We are crediting him, nevertheless, with having invented the theoretical basis for the rationality of the praxis which was carried out. In contrast to the trial and error method prevalent in the Arts and Crafts.* The originality of his personal experience, what makes of method the *cause* of the three treatises as *effect*, was that such was the way in which things

had occurred to him[14]. From such an experience he draws the conclusion that the same manner of explanations is bound to occur with other students of nature, once they study the three treatises as *examples* of how things can be explained. *I could not employ the use of the method (presented in DM2) in the three treatises because it prescribes an order in the search of results which is quite different from the one I thought should be used for explaining them*, he confides to his former teacher Vatier (see footnote 7). What, then, is the order that must be followed to provide an explanation? An example of such order is provided in a long letter (March, 1638) to an unknown correspondent, for whom (it appears from the text) Descartes felt considerable personal and philosophical respect. In paragraph 11 it is said: *It is well known that I do not pretend to persuade anyone that water is made of particles having the shape of certain animals, but merely that they are long, united, adjustable* (longues, unies et pliantes). *Hence if one can invent some other* model (trouver quelque autre figure) *by means of which one can explain all their properties in the way it is done with these, I am in favour of doing it; but if one cannot, I do not see any difficulty in imagining them in this manner, or in any other, given the fact that they of necessity possess a figure, and that the one I propose is quite a simple one.* The model proposed to provide an explanation must be adequate to the properties experienced in the *explanandum*. Therefore, the argument continues, ... *regarding air, although I cannot deny that some of its parts may have the same shape, there exists enough evidence to show that not all can have it, otherwise it could not have the lightness it has ...it could not expand or condense by degrees with the facility it does (si aisément qu'il fait), etc.* The explanation through models is related to the *properties one is considering;* consequently the model invented from a particular set of properties is subject to change; a principle which brings to mind the dual character of light and sub-atomic particles in present-day physics. An argumentation that leads Cartesius philosopher and student of Nature to the conclusion that regarding light *I should draw no conclusion regarding its true nature, and that it will suffice with two or three comparisons which permit considering it in the most convenient way to explain all of its experienced properties*, (ils n'est pas besoin que j'entreprenne de dire au vrai quelle est sa nature, et je crois qu'il suffira que je me serve de deux ou trois comparaisons, qui aident à la concevoir en la façon qui me semble la plus commode, pour expliquer toutes celles de ses propriétés que l'expérience nous fait connaitres ...) *continuing to deduce all the others that cannot be so easily perceived. imitating, in such a way, the example provided by the astronomers, who despite the fact that their presuppositions are mostly false or uncertain, the fact that they refer to diverse observations they have made, permits drawing very true and solid consequences*. The models of the astronomers are adequate to save the appearances (*salva veritate).*

The Cartesian line of thought on what constitutes an explanation, rejects the Aristotelian notion of the nature of events, be they light, or motion, or heat. When pursued in its logical consequences this rejection leads not to reducing everything to the realities of the meso-cosmos (which would represent a pseudo-Aristotelianism), *but to establishing if reason is to gain full control of the situation, a new set of standards to which not only the micro- or the macro- but also our familiar meso-cosmos would be reducible*. This ultimate unified cosmos created through a *unifying*

measuring yard has to be a creation of reason itself. One warranting the objectivity of reality; one which considers Nature an object of cognition because it is described according to the new standard. *Such a standard cannot be but a mathematization of experience*[(15)]. A way to identify what is existent or not, (as space-time inert being) through its being *subsumable under a mathematical concept*. In analogy to the sense-objects of realism the criteria for establishing the reality of a particular space-time K_j is whether it can be *demonstrated* through an experimental isomorphism as falling under a mathematical construct K^*. If the test *demonstrates* the existence of K_j, then K_j is explainable through K^*; and, one can deduce for K_j relations not taken into account when *demonstrating* the isomorphism with K^*.

The relation of truth as *adequatio* from which realism takes its start has been totally reestablished. However, it is not the intellect that conforms to the object, but the object which conforms to a construct of intellect through an isomorphic test. Model and space-time object share the same relational structure. That is the epistemic criteria behind the power of the operational-definitions of Galilean physics. The senses play a role, not in providing the initial raw material out of which the K^* originates but, through the primary qualities of K_j providing a particular space-time extension of the construct. Such kind of experimental testing, carried out in chemistry, can be carried out by daily-life sensorial criteria as a first step for providing an identity for a particular stuff. However, the final *scientific* identity depends on highly complex mathematical constructs.

Experimental testing as criteria for granting to an *objective mathematized* Nature the right of citizenship to enter into an *adequatio* relation should not be a surprise since the 17th Century had already agreed that the secondary qualities as means for establishing the identity of objects frequently lead to error or outright contradictions. It is when knowledge originates in a testing procedure that objective truth is attainable and becomes scientific. That is why the Kantian *Metaphysical Principles of Natural Science* states in the Introduction that *In any natural science the degree of scientificity is proportional to the amount of mathematics* it uses. From such a general principle on what constitutes a science he concludes that *As long as in the chemical reactions between elements of matter no model* (kein Begriff) *can be constructed that is, no law of attraction and repulsion among the particles* (Gesetz der Annäherung oder Entfernung der Teile) *can be formulated ... chemistry must remain nothing but a systematic art, an empirical doctrine* (systematishe Kunst, oder Experimentallehre) *never an authentic science since its principles are merely empirical* ... The absorption in our century of the theory of chemical reactions by the physics of particles has validated the Kantian point of view. Moreover, it should be noticed that when stating that no law of attraction and repulsion among the particles of matter involved in the chemical reactions is known, he is explicitly saying that for such a case Newtonian Mechanics is not applicable. Thus contradicting the current interpretation of Kantianism *as committed to maintaining the validity of Newton's laws of gravitational attraction for all interaction between particles of matter.* A serious blunder seriously affecting our understanding of the Kantian theory of scientific knowledge; or grasping its having been in significant aspects anticipated by Cartesius. (With certain advantages on the side of the latter as we shall argue,

below, in Section III of this essay). With the exception of the explicit discussion and explanation, on the part of Kant, as to how geometry becomes applicable to the empirical space of physics. An explanation consisting in the anticipation, on the part of Mind, of space *as an a priori pure form of our sensible intuition; which is what permits the application of Geometry to spatial events*. Once certain misinterpretations regarding Kant's Critical Philosophy are removed, a parallelism between the systems of Kant and Cartesius sheds light on the basic epistemic differences separating empirical-rationalism from naive realism. Let us briefly call attention to some common elements to Cartesianism and Kantianism, among them a reversal of the serial order of the elements in the cognitive process of realism.

1.5. Cartesius and Kant on the possibility of knowledge. Exactly 150 years after *DM* appears explaining how Mind, on the basis of its epistemic and mathematical constructs, combined with sensorial data, invents the possibility of attaining certain knowledge of space-time events, Immanuel Kant publishes (1787) the Second Edition of his *Critique of Pure Reason*,[(16)] in the Preface of which he makes a most startling revelation regarding the possibility of physical science, and of our knowledge of such a possibility -two problems also dealt with in *DM*, the former being the cause for publishing a perspicuous analysis of the latter[(17)]. The revelation consists in the *explicit* admission on the part of Kant that the method he used for establishing the possibility of the metaphysics as a science (the subject-matter of *Prolegomena*) *is warranted through being the very same method Mind followed in the invention of geometry by Thales and experimental physics by Galileo, Torricelli and Stahl.* Applied in each case to a different content: *The examples of mathematics and natural science, which by a single and sudden revolution in our manner of thinking developed into what they now are, appear to me sufficiently remarkable to prompt me to consider what may be the essential features of the new point of view which made possible such success.*(B xv) After a few considerations the idea of reversing the serial order of the elements constituting cognition makes its appearance: *Up to now it has been assumed that our knowledge of objects must conform to the objects must conform to the objects known ... Hence we should inquire whether greater success in the tasks of metaphysics does not ensue if it is the objects that should conform to our knowledge* ... (B xvi). In applying such a reversal to elements of cognition the author of the Critique is fully conscious of proceeding along the epistemic model that mouthed in Galilean Physics; the ensuing foot-note begins: *Such a method, modelled upon the one practiced by the students of nature, consists in searching for the elements of pure reason in what can be confirmed or refuted through an experimental result.* (B xix). Expressed in this passage announcing the reversal of the method of realism, (reversal providing the guiding thread to the whole Critique) is not only that the experimental method is common to all cognitive applications of reason. It also confirms his general view that the process of knowledge is an invention of reason for attaining its own aims: *For reason only has insight into what it develops following its own blue-print* (B xi) is the conclusion reached after examining the case of Greek geometry and Galilean physics. It also makes known his decision to apply to the process of knowledge as a holistic whole, the method used by mathematics in the case of each of its constructs. In the vocabulary of the Cartesian discovery of mathematics as pure form, Kant, through reflecting on the grounds from which the certainty of

mathematics originates namely, through being *constructions* of reason (B 741), also arrives at the idea of a *Mathesis Universalis*. Moreover, the unity of reason becomes present also in the scenario of Metaphysics: ... *Such an attempt to alter the procedure traditionally employed in metaphysics, thus revolutionizing it, through following the example set by geometers and physicists, constitutes, indeed, the main goal of this critique of pure speculative reason*[18]. A clear epistemic convergence (when not outright coincidence) with Cartesius, of which the following points are worth noting, appears:

a) If one is to attain certainty in the knowledge of the world, an alternative to the way realism conceives the process of cognition of reality is required. This alternative may well consist in reconstructing, under the *priority* of reason, not of the senses, the *epistemic* (not the psycho-somatic) process of cognition.

b) The elements constitutive of knowledge judgements namely, those originating in understanding, and those in sense must be preserved as constituent parts of the final product.

c) Through attributing to the elements contributed by the understanding an *a priori* character Kant is merely accepting the Cartesian position that Mind as a *res cogitans* separate from any sensorial content carries the responsibility for attaining *scientia*. In other words, in both systems cognition assumes the existence of *a priori* syntactic forms free from the contamination of sense-data.

d) The Kantian *a priori*, not originating in sense, provides the epistemic certainty equivalent to the Cartesian notion of the clear and distinct perception of truth. Mathematical constructs are *a priori*, and beyond doubt, respectively in each system.

e) The subsequent introduction of sense-elements into the cognition moving-belt of each system is done under conditions that do not affect the initial truth, warranted by Mind, that initiates the process. Finally we find in both a basic syntactic element, not originating in sense-data, which permits us to see them a different branches of the same epistemic tree namely, one originating in Mind.

f) The reconstruction of the cognitive scenario by the intellect not only reverses the serial order of its elements placing sense-data in a less prominent role than in realism; the action which takes place follows, in all its parts, a plan of reason acting as judge. The intellect becomes the judge of its own capacity for conveying certain truth with regard to nature. Cartesius would argue that *reason only produces judgements* (the *cogito*) *into which it has clear insight*. From which follows that what is seen as true can only lead to subsequent and new truth clearly perceived. Otherwise reason would be contradicting itself in its own game; The construction of truth.

The convergence of the Cartesian and Kantian systems represented in the above six points should be no surprise given their common epistemic goals namely, overcoming skepticism and validating the mathematical-experimental method as trustworthy, *rational and systematic*. A means for discovering the laws of nature. Both are bent on explaining how truth, whatever the subject-matter, can be attained;

an enterprise tantamount to asking how metaphysics, as the physiology of reason is possible. A considerable advance in our understanding of Cartesius and Kant becomes possible if we can provide an epistemic, not a psychological answer, to the problem: Why do they converge? When Descartes, asked by a visitor about the books he studied pointed to a carcass he was dissecting, uttered *Behold ! These are my books!* he was indicating not his interest in lambs, birds or cows but in organic structures relating the parts to the whole; structures in which the component parts become explainable in terms of the purpose of the whole; he was, metaphorically, saying: *When it comes to inquires regarding the concepts, ways and principles through which Mind operates in the life of truth one must look upon them holistically; as parts of an organic whole. In the case of knowledge one must separate sensorial from purely intellectual and mathematical components, which are subsequently organized for revealing the truth about nature in the way a clock is a lifeless artifact organized by his maker to measure time correctly, i.e. according to the purpose of its manufacturer. That is , one must find the dictionary where, for Mind, its principles and powers can be perceived and explained.* The most direct information regarding the grounds for the convergence of Cartesius and Kant is given in their writings when they indulge in intellectual autobiography; *e.g.* when Kant tells us it was Hume's skepticism that awoke him from his dogmatic slumber prompting him to ask whether Hume's general argument against the rationality of the principle of causality does not have a validity beyond that particular case.[(19)]

It is, consequently, a shared interest and faith in the powers of Mind to attain *scientia*, combined with a resolution to overcome skepticism and realism what explains the convergence of those two great thinkers. They also share the notion that whatever the powers of Reason, and wherever they can be detected in their operations the key to gaining insight into them consists in applying a step-by-step method adequate to speculative reason's *holistic* nature. This holism is emphasized by Kant not only in *Prolegomena*; it is also referred to on three occassions in the above mentioned Preface to the 2nd edition of the Critique, v. g. (B xxxvii) *Pure speculative reason exhibits an organic structure; one in which the whole is for the sake of each part, and every part for the sake of the whole* ... Such emphasis on holism appears when he has already become aware that geometry, Galilean natural science, and his own metaphysics constitute three examples of different sciences resulting from applying the same method to a different subject-matter. Cognitive success is attainable when Mind itself determines the conditions under which a functional relation both as to its form, as well as to its contents, expresses nature's behaviour.

Despite the structural similarities between Cartesianism and Kantianism as two varieties of rationalism significant differences originating in the historical circumstance under which each system appears become inevitable. Kant's Critique confronts a corpus of existent physical science. Consequently, its aim is to explain how such a science is possible. However, through employing the Aristotelian Table of Judgements as a master column of his philosophical edifice. he remains close, much to close to the traditions of the Schools. Cartesius on the other hand, became, since his years at La Flèche, deeply impressed by the certainty of mathematics compared to the fallibility of the senses and the inventive fruitlessness of the Schools.

Hence, his mission becomes establishing that based on the certainty of mathematics a new science of space-time events can be developed. *Je sais bien aussi qu'il pourra se passer plusieurs siècles avant qu'on ait ainsi déduit de ces principes* (the ones he has adopted) *toutes les vérités qu'on en peut déduire, parce que la plupart de celles qui restent à trouver dépendent de quelques expériences particuliéres qui en se rencontreront jamais par hasard, mais doivent être cherchées avec soin et dépense par des hommes fort intelligentes*... he states in the Preface to the *Principles of Philosophy*. Another conclusion to be drawn from the brief parallel between Cartesianism and Kantianism as epistemic systems is : What Cartesius discovers as the structure of a future science of space-time events leading to marking of man the master of nature has become a historical fact (the doctrine of progress) by Kant's time. The latter's philosophical mission, hence, is not anticipation but showing how Mind's *a priori* constructive activity is possible. The possibility of deducing the *a priori* concepts and principles warranting Galilean physics (including the limits of its legitimate use) constitutes what Kant calls Metaphysics as Science.

The critical limitations placed on the powers of speculative reason by Kant do not appear in Cartesius. For the Prussian, mathematics can be applied to experience because despite not originating in experience, ... *its concepts being constructions of Mind the propositions arising from them reach beyond themselves to what their intended intuitions contain*[(20)]. Mathematical concepts are endowed with inbuilt reference to potential space-time realities in a manner analogous to that in which our sense-perceptions are endowed with the capacity to provide meaning for our innate daily-life linguistic structures. When analyzing the epistemic scenario of the *adequatio* theory of truth the mind-body problem immediately becomes a mayor actor. The Kantian position to the effect that *the nominal definition of truth as consisting in knowledge being adequate to its object* (B82) implies that the agreement between object and intellect may occur in either of two ways to attain an *adequatio re et intellectus*: We start with *res* or we start with *intellectus*. Faced with such an alternative Cartesius initiates the long series of thinkers that choose the latter as conducive to an epistemology that fits the facts of science. Whatever the disagreements between Peripatetics and rationalists both stick to the *adequatio* criteria. *The historic quarrel has the appearance of being more about the language to-and-in-which the adequatio criteria is applied than about the reality we live in; or if the different languages we employ are Weltanschauung conditioned.* Cartesius would argue that Mind *following its own rationality* imposes a notion of space-time taking distance from their sensorial nature and making of them its own constructs; not as given *independently* from the cognitive process. Such is the meaning of the statement by Mersenne to the effect that *One is constrained to acknowledge that man is nor capable of knowing the reason for anything other than that which he can make; not other sciences than those of which he makes the principles himself, as one can demonstrate in considering mathematics*.[(21)]

Regarding the Kantian limitations for attaining truth, for Cartesius his demonstrations regarding the existence of God and the existence of a *res cogitans* and *res extensa* dualism constitute truths as solid, if not more so, than those of mathematics. For both thinkers, however, mathematics represents *the cognitive* gate leading to a broader philosophical scenario: For Kant to the realm of the *a*

priori; for Cartesius to a *Mathesis Universalis*. However, to arrive at such a conclusion Descartes, anticipating our own 19th century, had to discover the formal-relational structure of mathematics. To prepare our mind for such a paramount event relating to Cartesius and the philosophical context of the 17th Century (the manly century as A.N. Whitehead calls it) let us take a brief look at John Locke regarding the relations of mathematics to space-time events.

1.6. Locke on Mathematics and their physical application. That Locke's *Essay Concerning Human Understanding* bears witness to a rationalist (not a sensualist *tabula rasa*) concept of Mind was already asserted by Leibniz in his running commentary on the Lockian text. In the Preface to the *Nouveaux Essais*, not without a certain touch of irony, Leibniz says: *Perhaps our perspicuous author's own way of thinking is not too distant from my own* .... The basis of such epistemic convergence originates in an agreement as to the nature of mathematical knowledge *Our knowledge of mathematical truths is real because it refers exclusively to our ideas despite the fact that perhaps we nowhere find an exact circle* constitutes a Lockian assertion which all brands of rationalism consider adequate to their view of Mind as an active entity in contrast to the passive one of sensual-empiricism.Let us, in order to better appreciate the reasoning behind the Leibnizian remark, examine the Lockian *Essay*, Book IV, IV.6. regarding the role of mathematical constructs in our understanding of the world of physical objects. For the sake of clarity in our exposition the original text shall be divided in five sections *a, b, c, d, e,* (in italics) preceding, in each case, our own brief commentary (in plain text).

a) *I doubt not but it will be easily granted that the knowledge we have of mathematical truths, is not only certain but real knowledge ; and not the bare empty vision of insignificant chimeras of the brain, and yet, if we will consider we shall find that it is only of our own ideas.* In other words mathematical constructs, a product of Mind's creativity, do not originate in sense experience. Such constructs are not mere products of the individual's imagination; they possess a being which is both objective and rational. From this follows that,

b) The mathematician considers truths and properties belonging to a rectangle or circle only as they are in an idea in his own mind. For it is possible he never found either of them existing mathematically, i.e., precisely true, in his life. The mathematical constructs of Mind despite not originating in sensorial experience possess a reality as objective as that found in sensorial events outside the individual. To deny that Mind can validly report about intersubjective entities perceived through its own eye opens the door to Berkleyian subjectivism in that only the senses perceive whatever the subject experiences. With the consequence that no *adequatio* between mathematical constructs and space-time realities being at all possible. Galilean Physics becomes mere fiction. Locke asserts both the reality and the applicability to space-time events of our geometrical constructs. Whether a construct *R* applies to the space-time event *S** is a matter of experimental truth. (In this specific issue Cartesius differs as to the *how* of the applicability since he conceives of mathematical constructs as mere relational entities, *originally* devoid of any inbuilt empirical reference (Hilbert's implicit definition). It is this contentless conception of mathematics that leads to the invention of Analytic Geometry as will be shown in

Section II, below.) The applicability to space-time events results from a common relational structure, *i.e.* from the possibility of an isomorphic relation between a construct and a space-time event. Obviously the applied concept, in the process of knowledge, is *a priori* with respect to the event to which it applies[22]. In so far as a construct may apply to many objects it has an *a priori* intentionality. Contrary to the case of empirically induced or abstracted concepts in mathematics the intension precedes the extension. The *intension* of the construct, determines its extension *in relation* to space-time objects *a posteriori*, *i.e.* through an experimental validation-test applied to each specific case. In other words, mathematical constructs though exhibiting relational properties, have only *potential* extension. Which becomes actual through having reference to space-time events exhibiting an identical relational structure (Kepler's elliptical orbits). Through such constructs a new type of truth relation to reality emerges one *intending* a certain set of particular objects.

c) *But yet the knowledge he has of any truth or properties belonging to a circle or to any other mathematical figure are nevertheless true and certain, even of real things existing because real things are no further concerned, nor intended to be meant by any such propositions than as things really agree to those archetypes in his mind.* How do we rationally explain that a mathematical figure may be true and certain of real things existing? Given the fact that as has been established in a), these figures have existence only in our own mind? *The Lockian doctrine assumes as a fact that such agreement is possible, yet does not explain how it is possible.* Such a question regarding possibility receives an answer 110 years later through Kant's teaching regarding space and time as *a priori* forms of sensibility. Mathematical figures constructed by the mind, *anticipate the form of real events*, because Mind contains space and time as *a priori* forms of external phenomena[23]. However, an experiment does not have as its purpose to generalize a particular truth *but to verify that a given space-time event corresponds to a particular example of a theoretical functional-relation.* From the above Lockian schema explaining what should be understood by the applicability of mathematics to external phenomena necessarily follows that *there* is no such thing as *inductive* reasoning. For inductive reasoning demands the possibility of a falsificatory instance, *a possibility excluded, in principle, from the Lockian scheme.*

d) *Is it true of the idea of a triangle that its three angles are equal to two right ones? It is true of a triangle wherever it really exists. Whatever other figure exists, that it is not answerable to that idea of a triangle in his mind, is not at all concerned in that proposition. And therefore he* (the mathematician) *is certain all his knowledge concerning such ideas is real knowledge : because intending things no farther than they agree with those his ideas, he is sure what he knows concerning those figures when they have barely an ideal existence in his mind will hold true of them also when they have a real existence in matter...* On the whole the Lockian theory concerning the applicability of mathematical constructions to space-time events has a 2-stage structure. In stage 1, Mind has at its disposal a construct *K* with empirical reference; in the second it experimentally applies it to some indefinable space-time event S_j receiving a positive or a negative reason as to whether S_j was intended by *K*, In case an experiment shows S_j was intended by S_j, then S_j exhibits all the properties holding for *K*. The method is specially fruitful when K its what

modern physics calls a 'theoretical model', say the Bohr atomic model; in such a case through applying Bohr's model one can predict certain spectral lines ... until a new model is invented which covers cases not intended by Bohr's model. It is from constructing new theoretical models intending a broader field of phenomena that experimental results lead to a continuous progress in physical science. consequently, as a conclusion, we are told,

e) '*his consideration being barely of those figures, which are the same wherever or however the exist.*' If an investigator asserts that the relations R^* which apply to an object O_j, apply to O_{j+1}, and such turns out not to be the case, it is neither O_j, nor O_{j+1}, nor R^* but the personal expectations of the investigator Professor John Doe that are at fault. It has been a great epistemic error to apply to the logic of mathematical constructs the generalization aims of the logic of daily-life concepts. Such a lack of a rigorous epistemic differentiation between the *a priori* nature of geometrical constructs and the *a posteriori* one of daily-life concepts gives rise to a mistakes view of the true epistemic nature of the results attainable through the mathematical-experimental method of Physics. It has been a mistake to believe that the notion of a law of nature taken from a substantialist, daily-life view of space-time phenomena can be applied to regularities originating in the constructs of mathematics without provoking great intellectual confusion. What the Lockian exposition shows is that with the new method a different notion as to the logical and epistemic structure of a law of nature becomes necessary. From that, a new view of nature as an object of cognition follows. What in its turn requires a different view of Mind. One capable of accounting for, and acquiring a knowledge appropriate to such a new reality.

The difficulties inherent in playing a new knowledge-game the Lockian stand-point skillfully avoids by simply establishing that one cannot play the circle-game with a square, and viceversa. For the simple reason that the former does 'not intend' the latter. In similar fashion one cannot apply the experimental principles and conceptual structures proper to mathematical constructs to a qualitative reality as presented in the ontology and the logic of Aristotle. In other words, according to Locke, through mathematics what an experiment demonstrates is that some space-time event K_j is a particular case of a mathematical construct K^*. To how many other, or what set of objects presumably forming the extension of the construct K^* it applies, is not predetermined through the intension defining K^*. However, the fact of the extension of K^* can only be determined through the relational structure defining K^*. It is through each particular experiment E_j that the extension of K^* becomes gradually known. Although each construct K^* has an intension prior to E_j it is through the E_j 's that the extension of K^* becomes quantitatively determined. Consequently, there is no possibility of an inductive falsification in the sense of naive realism. This futility of enumerative induction for scientific theories based on the use of mathematical constructs had already been clearly anticipated in Bacon's *Novum Organum*, 105: 'In establishing axioms another form of induction must be devised than has hitherto been employed... for the induction that precedes by simple enumeration is childish, its conclusions are precarious, and exposed to peril from a contradictory instance.'(24)

The Cartesian mathematical-experimental schema in contrast to the Lockian is not one establishing a contrast between sensorial realism and rational empiricism, but one between two varieties of the latter. A contrast meant to enhance the importance of the Cartesian ordering of the constituent elements (the mathematical, the empirical reference, and the experimental validation), in three instead of two stages as is done in Locke, in whose *Essay* the first two collapse into one. Through such a contrast the originality of the Cartesian discoveries become, once more, clearly demonstrated. Section II of this paper, anticipating the modern view of mathematical constructs as 'pure forms lacking in empirical reference' presents the Cartesian one of three stages. However, once the stage in which mathematical constructs attain empirical reference is reached, the process dove-tails into the Lockian schema.

1.7. Conclusions to section I. The method adopted by Galileo and his disciples regarding the relations existing between mathematical constructs, experimental validation, and space-time reality was also the method rigourously practiced by the 'incomparable Mr. Newton' whose close relationship to Locke in scientific matters is a well established historical fact warranting that *Principia* attains a systematic explanation in the Lockian schema of IV. IV.6. In broad outline its essential elements and stages in the praxis of Physical Science are the following: Mind has become aware of its own powers for investigating experience through an open-ended procedure which, contrary to realism, takes-off from the constructs of reason. It then devices experiments in order to provide extension to the intension of each of its constructs. The extension is the result of an *adequatio* relation of the constructed intension with objects of a specified domain. What is important is that the domain of validation may be any object, physical or mental to which an *adequatio* experiment may be carried out for which a truth-value can be defined through testing. In the case of Galilean physics the testing is of fairly simple space-time events, namely: One constructs a functional relation $F(x,y)$, *i.e.* one demonstrates syntactically $F(x,y)$ included its empirical reference; thus $F(x,y)$, a construct of the mind, becomes a potentially true functional relation. The next step is to demonstrate its actual truth[(25)]. That is done through an appropriate experiment, with some here and now space-time event. In case the result is positive, we have experimentally demonstrated the actual truth of $F(x,y)$. Such has been the praxis of Physical Science during the Scientific Revolution and after, despite the insistence on the part of sensorial empiricism that the construct $F(x,y)$ itself originates in observation. Such empiricism attributes to the experimental technique a generalization-inductive character valid only for the logic of concepts expressed in daily-life language[(26)]. In this respect, three centuries later, the creator of Relativity expresses an essentially identical view to that of Cartesius and Locke when saying: 'Every linguistic utterance is wholly confined to the linguistic sphere. Concepts, as far as they have any basis, are -judged logically- free inventions of the mind. (together with propositions connecting them).....There is no logical way to deduce concepts and propositions from our crude experiences ("induction")[(27)]. The Einsteinian position induces us to conclude that from Descartes and Locke to Einstein and Heisenberg the Philosophy of Physics, i.e., the meta-language of Galileo's mathematical-experimental physics has changed in three centuries, if at all, less than its content or object-language.

Thus revealing that the Metaphysics of rationalist-empiricism has an enduring value for the history of thought.

SECTION II

From Analytic Geometry to *Mathesis Universalis*

The Cartesian thought process mouthing in Analytic Geometry (henceforth *AG*) constitutes not only a significant contribution to mathematics. As outcome of an epistemic oriented non-mathematical reflection, is constitutes perhaps, a unique case in the history of thought. However, it would be a serious philosophical mistake to believe that because of its genesis in a non-mathematical problem namely, the quest for cognitive certainty, *AG* was the result of mere chance, a fortunate creative event no doubt; nonetheless an accident in the life of René Descartes. Like finding a priceless diamond when looking for one's fountain-pen in the family house-lawn, fortunately, Cartesius himself informs us of what was going on in his mind when *AG* appears in his philosophical scenario. The Cartesian *Eureka!* associated with the invention of *AG* emerges within a systematic and critical non-Aristotelian inquiry regarding the cognitive powers of Mind. The aim of which was to establish a rational warrant for the certainty of mathematics. A warrant of truthfulness powerful enough to reach into the new and controversial mathematical-experimental knowledge of space-time events obtained by the followers of Galileo Galilei. *AG* like all great discoveries that add some new and unsuspected dimension to our cognitive powers was hit-upon in what Kant calls a sudden flash affecting a single individual with enough force so as to cause a revolution of such power in our manner of thinking that the road that be followed in order to attain *scientia* from then on could never give occasion to a mistake (*der glückliche Einfall eines einzigen Mannes... von welchem an die Bahn die man nehmen muß ste, nicht mehr zu verfehlen war...*) (B ix), to conveny by means of a popular metaphor his own evaluation of the importance of such epistemic flashes impinging on a individual's mind, he continues: *The history of this revolution is to our manner of thinking more important than discovering the way around the famous Cape of Good Hope....* For the History of Philosophy it is worthwhile noticing that on this occasion, talking about epistemic flashes no mention is made of Copernicus, whose name only appears six pages further on. Once such an unexpected discovery is made (*AG*), the streak of genius of Cartesius consists in becoming aware of its revolutionary impact on our views regarding the powers and the ways of Mind. Such powers become the cause of an effect namely, providing what 19th century philosophical discourse refers to as the meta-language of, among others, our present-day mathematical-experimental science of space-time events. Cartesius, moreover, not only foresees major theory-building consequences of his insight into the formal nature of mathematics for a science of natural phenomena (including medicine). Anticipating the social impact ensuing from a methodic and rational capacity to manipulate nature made possible by his view of knowledge, he considers it his duty to make public his newly found vision of nature. However, mastery of the new vision required for carrying it into practice establishing these principles on a philosophical foundation, *... a matter which I could expect to accomplish only after attaining a greater maturity than that provided by the 23 years I then had, ... and only after investing a lot of time in preparing myself through*

discarding the false opinions received by me up to that moment (his personal flash of 20.XI. 1620) *as well as gathering more knowledge from a greater amount of experiences*... (*DM* 2). Kant's reading of whatever writings of Descartes he had at his disposal seems not to have provided him with enough insight into the epistemic importance accruing to the content and consequences associated with the Cartesian interpretation of his discovery of *AG*.

2.1. Analytic Geometry as an epistemic discovery. The discovery invention of analytic Geometry viewed in the context of *a general method of discovery, i.e.* one which can be applied to a variety of content (the *Dioptrics, On Meteors, Geometry*), leads to valuate insights into the epistemic originality of Cartesianism regarding the essentials of *how* and *what* Mind, *exclusively acting through its own powers*, is able to know. *AG* turns out to be, with regard to sense-data a higher, purer form of cognition than traditionally associated with Euclidean geometry; *AG* reveals a hidden cognitive structure of mathematics awaiting, like the vast American continent, its philosophical Columbus. The mathematical structure of *AG*, reveals itself exemplified in two familiar models : The geometrical plane; and the operations of Arithmetic. The discovery of a higher purely formal structure hidden beneath the surface of traditional mathematical disciplines although a novel epistemic event is interpreted by Cartesius as another step in Mind's road to *objetive truth*; inbuilt into language and its traditional logic of the Schools supporting the empiricism of cognitive realism. *AG* discloses, employing a one-to-one mapping, the existence of a single structure shared by two, until then, believed to be independently subsisting entities namely, Geometry and Arithmetic. Their *reduction* to a common structure is epistemically identical, with discovering -through a sudden mental insight- that two empirical circles with, say, different radius, are reductible to *particular* examples of the geometrical construct 'circle'. Or that four apples and four fingers share the quality of 'fourness'. In the case of *AG* the alleged independent subsistence of geometry and algebra was a function of applying criteria 'contaminated' by sense (through actual or potential applicability). A moot independence based on ignoring a '*higher*' construct to the extension of which they belong. Erroneously considering as *ontological* a difference originating in applicability.

2.2. A priori constructs and individual identity. In order to discover shared structures between entities (empirical or mental) judged as different, a criteria from which sensorial elements, actual or potential, are absent must be applied. As happens in daily-life when an object *O** maintains its identity at different time-moments, one might go as far as saying that the identity of *O** is a function of the language used to talk about *O**. In the case of *AG* a phenomena natural to Mind takes place in relation to Algebra and Geometry. The common structure of Algebra and Geometry was discovered independently by Fermat and Descartes. An isomorphic mapping from geometrical curves into algebraic expressions settled the matter. Hence from a mathematical, non-sensorial point of view, those two sciences are but one. Fermat drew no non-mathematical consequences. Viewed as an epistemic result, however, the different presuppositions lead to different consequences. *The interest of Cartesius in mathematics originated in their certainty as cognition; they provided an example of the powers of Mind*. If their certainty derives from their being pure form, albeit encoded in a different subject-matter

(Geometry and Algebra), the question arises: Can we not on the basis of the notion of *formal structure* develop a universal system of knowledge, a *Mathesis Universalis*, a science of pure relations? Such a science using the constructs of mathematics would make possible a new type of *objectivity in our cognition*. A truly universal science. A new paradigma not within, but *defining* knowledge invented by Mind? May it not be the case that from the point of view of Mind *all reality* shares a common feature? That when through the senses pure form reveals itself? A rational integration of *res cogitans* and inert matter would be solidly founded. *Pure form would then be the common denominator to all that the senses reveal. The radical irreductibility of sight to smell or sound would be eliminated through the common denominator*. In other words, are not the sense qualities merely the way in which, as organic beings, we perceive what to the keener eye of Mind is, in reality, a bundle of relations? The science of these bundles being what constitutes mathematics.

If the above Universal Science of pure structures were possible, then, by means of a dictionary of pure forms combined in different ways, individual possible worlds could be invented *a priori*. the cognitive problem would be reduced to inventing, *for each system* of *a priori* constructs, a mean for determining which is the real one. Such a means for selecting the existing world follows directly from the fact that *any one of the a priori worlds is but a system of relations*. Which of these systems holds for our space-time events becomes a matter of observation. Such acts selecting the real from the merely possible is the role of experiment. The experiment E_w revealing which of the possible worlds W_k is the real one is, however, not a matter of one hour, a month, a hundred or a thousand years. It is coextensive, in time, with the History of Physics. What a Nobel Prize winner obtains as recognition for his inventiveness and minuteness corresponds to a mere fragment of E_w. What an experimental physicist does is to take an isolated fragment from a given mathematical system, subsequently testing, through operational measurements, its relation to space-time reality.

Such is the role played the experimental technique in a scientific (mathematically expressed) view of the world. Moreover, the criteria used for establishing the identity of an object is (using a relational-quantitative vocabulary) the same used in Aristotelian logic: An object is classified as a type of being (with genus and species) when is has certain notes (logic of intension and extension of concepts). The difference with Cartesian logic is that the latter uses mathematical relations to identify individual objects. Such is the epistemic meaning of the mathematical-experimental method. The name itself explains what it's all about. In the epistemic mutation from Aristotle to Cartesius, Mind, indeed, changed its cognitive vocabulary; but not the operations and aims it accomplishes. The mistake of the Schools is to assign to the cognitive categories originating in the Table of Judgements an absolute, exclusive, all-embracing value; the fact that they refer to an specific cultural anthropology notwithstanding. The acknowledgment of such a fact suggest adopting a different set of principles.

Believing to have found such a new set Cartesius declares (Preface to the *Principles of Philosophy*): *Ce sont là tous les principes dont je me sers touchant les choses*

immatérielles ou métaphysiques, desquelles je déduis très clairement, ceux des choses corporelles ou physiques..... L'autre raison qui prouve la clarté de ces principes est qu'ils ont été connus de tous temps, et même reçus pour vrais et indubitables. His own originality does not originate in admitting the truth of such principles but in putting them to a different use. One related to the coherence of a method and of a system different from those of Aristotle : *Mais encore que toutes les vérités que je mets entre mes principes aient été connues de tout temps de tout le monde, il n'y a toutefois eu personne jusqu'à présent, que je sache, qui les ait reconnues por les principes de la philosophie, c'est-à-dire pour telles qu'on en peut déduire la connaissance de toutes les autres choses qui sont au monde...* . It is the stated purpose of *DM* to reconstruct the process which led to the discovery of a science of form, an *a priori* knowledge of *'les autres choses qui sont au monde'*.

Once new criteria for defining truth (from sensorial cues to formal structures) within the cognitive process is adopted -whatever consequence can be stated: Reconstructing the process of cultural anthropology undergone by geometry. An analysis of its historical origin in ancient Egypt reveals that our original experience is of perceived physical examples of triangles, squares, circles and other simple geometrical figures. A next stage in the ascent towards the modern purely formal conception represented by Hilbert's *Grundlagen* of 1899 takes place when Thales or someone else conceives definition. The triangle becomes *demonstrated* (a non-deductive process of mind as is traditionally misinterpreted). And in such a state it remains, up to the Cartesian invention of *AG* for more than 2500 years. The great Euclidean achievement is to show (demonstrate, invent) how, from a few postulates explicitly providing specific construction rules, a great many constructions follow (they become *demonstrable*). The Euclidean constructions, it is essential to take notice, are endowed with empirical reference. Cartesius, because of his paramount interest in attaining certainty, abstracts from the empirical reference; empties the Euclidean constructs of sensorial content. What is left? A Hilbertean formal structure! there is no reason to believe that such an idea of depriving geometrical constructs (as present in the Lockian scheme of IV. IV. 6 supra) of their empirical reference could not be entertained by some other individual in the course of History. Specially by one, who as in the case of Cartesius, had an interest in examining the powers of Mind as a substance independent from *res extensa*. That precisely is what he did according to the story told in *DM*.

2.3. From abstract forms to sense-realities. Once the process of abstracting from sensorial elements provides access to the realm of pure forms there is no reason why the serial order followed in said process cannot be reversed. Specially when considering that relational structures are neither a question of sense data, nor of the imagination. Through a reversal the gradual deprivation of sensorial elements running from total to zero content, becomes transformed into a *filling-in one*; running, this time, from zero to 100% sensorial content. The serial -order of the filling-in process can be expressed, in simple Galilean fashion, through the question: Given an empirical event E_k, to which pure form F_j does it correspond? With such a question the process of cognition, becomes broken down into three, not as classical positivism postulates, two stages: The first stage is pure form; the third is sensorial experience. The key to Galilean experimental physics lies in stage

2, acting as epistemic bridge between stages 1 and 3. Under this 3-stage process, stage 2 represents the possibility of stage 1 considered as the pure form corresponding to an empirical content. In the case of pre-Cartesian Arithmetic and Geometry, these two disciplines were endowed with empirical reference. The student of Nature had to think them in terms of spatial forms or of counting objects, in order that a given space-time reality be taken as their experimental validation. From among all possible worlds physical science has selected a specific on which has systematically emerged, through the history of Physics, as the real one. *The process of filling-in mathematical forms is a continuous one with no visible end in sight.* [(28)]

Among the significant events made perceptible through a Cartesian reading of the Cartesian text it is worthwhile highlighting how an epistemological non-mathematical reasoning. Such an application leads to differentiating between what we now days call *theoretical and applied mathematical models*. Also to the separation between syntax and semantics (form and content) as differentiated elements in the judgements constituting Geometry. It is precisely while paying attention to the epistemic significance of such *differentiated operations of Mind* that Cartesius discovers the existence of a purely formal stage in a geometrical system. Anticipating in 150 years the revolutionary insight presented in Hilbert's *Grundlagen der Geometrie,* the insight out of which the system of *metamathematics* evolved, such an anticipation of the formal character of mathematical constructs originates in the Cartesian *determination* to doubt any truth relying on sensorial data for its certainty. From such a decision follows that the certainty of *geometry requires that it be construed as a system of pure relations* uncontaminated by any *actual or potential* sense reference. The realm of pure relations opens a new era in the history of knowledge. *Introduces a new epistemic paradigma, not simply one in the object-language*. Let us focus our attention on the presentation in *DM.*

2.4. Analytic Geometry in the text of Discourse on Method. The above epistemic revolution takes place, the story goes, one day when as *I was returning to the army from the coronation of the Emperor, the setting in on winter arrested me in a locality where, as I found no society to interest me, and was fortunately undisturbed by any cares or passions, I remained the whole day in seclusion, with full opportunity to occupy myself with my own thoughts*. In the midst of this reflective mood the possibility of using his mind correctly and fully becomes an intellectual goal and sort of moral compulsion. During his years at La Flèche young Cartesius had been greatly impressed by the precision and clarity of mathematical reasoning in contrast to *the towering and magnificent palaces with no better foundation than sand and mud* built by the ancient Moralists: *I was specially delighted with Mathematics on account of the certitude and evidence of their reasonings.* The fact notwithstanding, he says, that *I had not precise knowledge of their true use; and thinking that they but contributed to the advancement of the mechanical arts, I was astonished that foundations, so strong and solid should have no loftier superstructure reared on them.* (*DM*1) What begins as a psychological feeling (astonishment) heralds the hope that *when* properly analyzed and understood Mathematics *should have a superstructure reared upon them.* Such a loftier superstructure turns out to be mathematical-experimental physics.

2.5. Anticipating Hilbert's formal geometry. Reflecting on the certainty of Mathematics; obsessed with the idea that some basic laws to guide the mind in its search for truth must be found Cartesius reflects on the advantages and consequences of eliminating actual or potential sensorial elements from the demonstrations of traditional Geometry. The process is initiated with reflections favorable to the method of mathematics. convinced, we are told that *the mathematicians alone have been able to find any demonstrations, that is, any certain and evident reasons... not anticipating, however, from this any other advantage than that to be found in accustoming my mind to the love and nourishment of truth, and to a distaste for all those reasonings as were unsound ... I resolved to commence...* . Once that is done, like a hungry philosophical lion having at his feet a stain geometrical antelope that voraciously jumps upon what remains under the guise of a relational skeleton, *observing that, however different their objects (in the several branches of Mathematics), they all agree in considering only the various relations or proportions subsisting among those objects. I decided it would be better for me to consider those proportions in their general aspect...*

Reference to the sensorial objects of the mechanical arts once cast aside, insight into the realm of pure relations ensues : *I thought it best for my purpose to consider these proportions in the most general form possible, without referring them to any objects in particular.* In the next step, Analytic Geometry appears: *In this way I believed that I could borrow all that was best both in Geometrical Analysis and in Algebra, and correct all the defects of the one by the help of the other.* Perceiving how a specific problem originally expressed in the language of one of these two disciplines could be better understood by translating in into the language of the other thereby attaining a solution, his own mastery over outstanding problems in the History of Mathematics was greatly increased. It was, hence, not mere illusion to believe that what really had been found was a more general and deeper insight about the cognitive process warranting its application in areas lying beyond mathematics, yet sharing their certainty. A certainty which originated more in the method employed with its roots in Mind itself than in the specific subject-matter dealt with. *In point of fact, the accurate observance of these few precepts gave me, I take the liberty of saving, such ease in unraveling all the questions embraced in those two sciences, that in the two or three months I devoted to their examination not only did I reach solutions of questions I had deemed exceedingly difficult, but even as regards questions of the solution of which I continued ignorant, I was enabled, as it appeared to me to determine the means whereby and the extent to which a solution was possible.* So much as to content. However, viewed as an advance in the quest for certainty what has been discovered is a *shared purely relational structure* permitting dealing *à la* Hilbert, *i.e.* without reference to specific content, whether geometrical or arithmetical, with problems stated in any one of the specific domains. With the benefit of hindsight we perceive that what Hilbert's implicit definition system does is to establish a system of game-playing with linguistic constructions originating in reason so that the problems that appear in it never abandon the formal scenario. It is a system of self-reference of reason with itself. Consequently, in so far as it never abandons the domain of reason, it has universal validity when dealing with its own constructs.

As a result of such subtle epistemological differentiations Cartesianism impregnates philosophy with and epistemology totally at variance with the philosophy and logic of the schools. The *nihil ist in intellectu quod prius no fuerit in sensu* of realism (based on the *tabula rasa* model of mind combined with a substantialist ontology) is replaced by *nothing in space-time reality can be mastered by reason unless it is identified as a particular case of a construct of reason.* It follows that whoever controls the world of relations becomes, in the course of time, master of reality; or, in first instance, of the scientific vision of it. Science being, precisely, a method for expressing objects and instances of a relational structure. The requirement for attaining success originating in the fact that these relations have intersubjective validity, are independent from any subjective particular sensorial apparatus perceiving reality. They cannot be based on the senses. *Neither onto- nor filogenetically*. Who can doubt that objects are given to us within a relation-complex? This is the origin of the new method for envisaging natural phenomena for *the book of Nature is writ in the language of geometry* as Galileo proclaims in *Il Saggiatore*.

Proportions and relations applied to space-time phenomena not only allow measurement. They demand it. Quantity is the language supporting the science of reality. The innovative creative aspect of the whole new method, causing bewilderment, social unrest and political and theological suspicion, is that *relations are neither perceived nor spontaneously found in Nature.* They are produced by Mind, *a priori*; once demonstrated as a construct of Mind they demand to be *experimentally* demonstrated through observable phenomena as was currently experienced in Music, Astronomy or Physiology (Dr. Harvey); in civil Society as is the case with *Leviathan*; or in the History of Civil Nations in Vico's *Scienza Nuova*. The Cartesian train of thought continues: *Results attributable to the circumstance that I commenced with the simplest and most general truths, and that thus each truth discovered was a rule available in the discovery of subsequent ones.* In *Rules for the Direction of the mind,* Rule V, he states: *Method consists entirely in the order and disposition of the objects towards which our mental vision must be directed if we would find out any truth. We shall comply with it exactly if we reduce involved and obscure propositions step by step to those that are simpler...* . It would be a serious mistake to interpret these rules of method as establishing psychological rules for effective learning. The Cartesian system is not despite the resemblance of vocabulary concerned with the psychology of learning. A circumstance which makes insight into its epistemological depth truly difficult. A difficulty, on the part of the reader, prompting Cartesius to state in the Epistle Dedicatory of the *Principles of Philosophy* to Princess Elizabeth, regarding his writings: *Car it y en a plusieurs qui les trouvent très obscures, même entre les meilleurs esprits et les plus doctes.; et je remarque presque en tous que ceux qui conçoivent aisément les choses qui appartient aux mathématiques en sont nullement propres à entendre celles qui se rapport à la méthaphysique, et au contraire que ceux à qui celles-ci sont aisées en peuvent pas comprendre les autres.*

SECTION III

Cartesianism in the III Millennium

'Nonetheless, as it is not from the roots or the trunk of a tree that one culls the fruit, but only from the extremities of their branches, so the greatest benefit to be derived from philosophy depends on those things we cannot learn till the end' (Preface to the Principles of Philosophy).

A few conclusions relating to present-day philosophical discussion may be drawn from our essay by asking a simple question inspired in a Cartesian metaphor: *Which fruits may we expect to be culled from the Cartesian philosophical corpus relevant our civilization ad-portas of its third millennium ?* This question may be adequately answered on condition of previously passing review to a roster of themes affecting our present cultural daily-life scenario, as a way of selecting three with roots in problems dealt with by Cartesius. Applying such a criteria the list of possible themes is bound to include almost anything making headlines in the media; with the possible exclusion of what has reference to world peace, the 'demise' of marxism-leninisn, and the Palestinian-Israeli conflict. Cultural and economic globalism; ethnic conflict; overpopulation; the gradual marginalization and increasing public indifference to the liberal-humanistic values; ecology; drug-abuse; economic, social and personal insecurity; ethnic and religious fundamentalism; the control and institutional uses of medicine; the rapidity of technological change, and many others themes affecting our vision and apprehensions regarding the future of post-industrial society, are clearly related to the philosophical and humanistic legacy of Cartesianism. Consequently, I beg to be allowed to bring to an end this essay by briefly referring to three Cartesian themes succinctly dealt with in the first two sections.

The selected thematic trio consists of the following:

a) What is meant by a method or logic of invention?

b) The neuroscience debate against Cartesian dualism.

c) Mathematization of nature, and daily-life reality.

Within the holistic perspective required for serious philosophical debate in general, and for Cartesianism in particular, it is clear that the components of the trio are thematically interrelated. However, despite the fact that each one belongs to some specific domain of knowledge, none is considered as pertaining to the exclusive interest of a small group of specialists. The same happens with other major topics of discussion among specialized groups which, despite their narrow technical connotation, constitute an integral part of the meaning of modernity. As the historical process since the Renaissance, giving birth to modernity, is institutionally interpreted by western mentalities. By omitting any reference to the order in which the selected themes are interrelated and abstruse philosophical discussion subject to prolonged debate can be, I hope, avoided.

Theme a) permits us to present the position assumed by Descartes when his Parisian correspondent Marin Mersenne, together with a group of theologians, towards the end of the Second Objections to the Meditations, recommend presenting them *according to the method used by the geometers, so that by one single glance at the text* (tout d'un coup) *they could be understood.* Such a request elicits an answer in which paramount clarifications as to the way Mind operates *both in the search as in the method employed for expounding truth* are made : *It is worthwhile to take*

notice here of the extent to which I have followed the method you recommend and the extent to which I plan to follow it in the future. There are two factors to be distinguished in the geometrical mode of writing, viz. the order and the method of demonstration. The first element has to do with content, but not in relation as to how Mind has gained insight into it, but as to how it is related to other contents. Therefore he explains, *Order consists merely in presenting what one has in mind so that the first things may be grasped without having recourse to what follows; hence, presenting what follows in such a manner that their demonstration depends solely on what precedes them. This procedure I faithfully tried to follow in my Meditations; and it was for that reason that I dealt with the distinction between the mind and the body not in the second but finally in the sixth. In the Meditations I also deliberately omitted dealing with many things when it was the case that they presupposed explanations of much else besides.* The similarly of the order followed to the deductive method of Euclidean Geometry or in formal game playing is clear. And *that,* precisely, accounts for the formal coherence and order traditionally associated with *more geometrico.* However, *the problem of intersubjective meaning and truth and the way they are communicated corresponds to a different operation of mind.* One in which and objective judgement is shared by a plurality of psycho-somatic minds. Without such and epistemic substitution of biological minds by an intersubjective epistemic Mind the Scientific Revolution becomes unthinkable. At this point the term *demonstration* acquires and epistemic connotation different from 'formal deductibility', or from being the 'order in a deductive chain': *The manner of demonstrating, we are instructed, is a double one: either by analysis or resolution, or by synthesis or composition.* After such a *distingo* a substantial explanation revealing the equivocal nature inherent to the current notion of a *geometrical method of demonstration* ensues; the epistemic complexity of the logic of invention supporting the Scientific Revolution becomes the subject of a carefully demonstration[(29)]:

Analysis shows in a systematic manner the true process through which something has been methodically invented so that insight into the way an effect depends on the cause can be gained; in a manner allowing the reader, if he so wishes, to understand what is demonstrated in such a perfect and immediate way as if he himself had invented it... However, such type of demonstration is of no avail with stubborn or inattentive readers, *Synthesis, on the contrary, through an entirely different method ...makes one see how the consequences are contained in the antecedents, forcing the reader to give his assent no matter how stubborn and prejudiced he may be.* In analysis, preceding from cause to effect a deeper understanding ensues because the individual listeners mind *retravels Mind's act of discovery*. Demonstration *shows* a relation between different things in a way analogous, in its immediateness, to an act of sense perception. There takes place a disclosure, an ostensive acquaintance with the object; a *monstrare,* in the Latin, intuitive, sensorial non-syllogistic immediacy meaning of the term. Only that it is to the intuiting individual mind that Mind's argumentation *demonstrates.* A sort of mental illumination. The ancient geometers, asserts Cartesius, in their writings were in the habitat of using synthesis, *not because they were ignorant of analysis, but, in my opinion, because they considered analysis so important that they kept it for themselves as an important secret.* As for himself, breaking -as a spin-off from his insight into the methods used

by the mathematicians-, the monopoly on Analysis held by the geometricians, he says *In the Meditations I have followed exclusively the analytic method because it seems to me trustworthier and the more apt for teaching... .* By considering theme a), not only is a better understanding of the Cartesian *method of analysis or resolution* attained. As important clarification as to what is meant by *demonstration*, in its non-formal, non deductive *use* during the 17th century is also provided[30]. By following Cartesius', analysis of how Mind works in its 'logic of discovery' *whatever the content* we arrive at the idea of a *Mathesis Universalis*, (or Universal Reason) a theme we shall say something about when dealing with Theme c). The initial criteria of *DM* demanding absolute certainty prior to giving assent to something is the same employed when *demonstrating through analysis*. It is the criteria warranting cognitive rationality and certain truth. It is, *reason and truth itself.* Two centuries after Cartesius these epistemic insights reappear in the Fregean doctrine of Mind through the universality of the functional relations. among them the non-predicative character of truth; the invention of sentential calculus with universal and existential quantifiers; and last but not least, providing existence with the logical status of a second-order *predicate,* in opposition to Kant's refusal to grant to existence a logical status.

Theme b) introduces the debate taking place at the interface between neuroscience and philosophy. Or, in the name of the former, against Cartesian dualism; a debate that has significantly increased in the last decade. The reason? Most practitioners in neurology and related activities believe that the *'reduction'* of the entity traditionally referred to as Mind, esprit or soul to processes definable in terms of brain anatomy, physiology, or chemistry (brought together in what Charles Stevens, of Salk Institute, calls *Theoretical Neurobiology*) constitutes a theme *exclusively* belonging to the domain of scientific research[31]. Consequently, philosophers, cosmologists and theologians of all creeds and nationalities should maintain a prudent distance (when not outright silence) acknowledging there is nothing 'philosophical' to it. Whatever can be decided on such matters should be done in the laboratory of science..., and not be discussed elsewhere in a non-scientific language. If we continue discussing pseudo-problems by referring to entities not expressible in the language of the physical brain it is because our scientific knowledge of how said brain functions is *hélas* incomplete. As a result of such insufficiencies we invent non-material causes for individual and cultural phenomena which because of our ignorance have not yet been reduced to *res extensa*. The person responsible for endowing with philosophical status our ignorance regarding the creativity of *res extensa* is the inventor of Analytic Geometry, the Frenchman René Descartes, born on the 31 of March in La Haye, a small town in la Touraine.[32]

Attacks on 'dualism' from different points of view were a favorite theme of the many and interesting lectures and panel discussions at the *Rencontre Philosophique* held (Paris, 1995) under the enigmatic and Cartesian collective name of: *'Qu'est-ce qu'on en sait pas'* programed by UNESCO on the occasion of celebrating the fiftieth Anniversary of its creation. A significant element of the reductionist dynamics originates in W. O. van Quine's monist slogan, *think epistemology as constituting a part of nature*. The attacks against dualism by no means approximating its final stage, open new avenues for philosophical debate; and, with the significant advances

in ethology, (inspiring, among others, Ruprecht Riedl's *Evolutionary Epistemology*) for a renewed interest in Descartes. Does the feasibility of mapping all knowledge into brain areas entail that dualism is a pseudo-problem? Is dualism committed to asserting that not all *scientia* can be mapped into brain topography? If an expression like 5+7=12 can be reduced to a brain process, how about attributing intersubjective truth to it ? Is such mapping as unthinkable as a square circle? Such, and other questions belong, it seems to me, to a *Problemkreis* deeply rooted in Cartesian philosophical thinking.

Theme c) engages the historical process through which the total mathematization of nature, as proclaimed by Galileo, became the guiding principle for all *hard* science. Kant takes account of this in a dual way: In relation to the nature of our intellectual powers; and by drawing the consequences as to what is validly attainable under such a concept of knowledge. The XIX century, aware of the different content and method involved in traditional scientific thinking developed the duality between *Natural and Cultural Sciences*. Apart from such a duality, and within the anti-positivist line of thought followed by rationalism, does the convergence of Cartesianism and Kantianism to a mathematization of reality mean that two systems have no significant differences?. Neither in their conceptions of how knowledge is possible nor in the consequences for providing a firm basis for overcoming skepticism, and modern reductionism? Such are questions that go beyond the aims of this essay, yet some brief tentative answers may be provided. There seems to be a parallel between the tree of knowledge and the bifurcation of reason into the three Kantian Critiques. Yet, the greatest difference lies in the explicit Kantian investigation of the *a priori* elements and principles required for applying mathematical constructs to nature. The problem not being how to acquire knowledge of nature; it is already present as a historical fact. What matters is its use and its theoretical and practical limitations. Thereupon what has to be carried out is an epistemic and cultural reconstruction of the process Mind undergoes in order to produce synthetic *a priori* judgements.

In the case of Descartes the contrast between synthetic *a priori* and synthetic *a posteriori* is a question of the origins of the concepts and the ensuing judgements, resulting from a quest for certainty. Certainty cannot originate in sense experience. Only reason can warrant in the manner it warrants the certainty of mathematics. Or the existence of the world we live in. The Sixth Meditation begins: *It is now time to examine if material things exist ... At least I know that they can exist in as much as one considers them as the subject-matter of the demonstrations of geometry*. Mathematics, however, in the Cartesian conception originating in the invention of Analytic Geometry as told in *DM* has no *per se* reference to world events. It is, like chess or poker, a game built on pure formal relations. *Mathematics ... considers only the relationships or proportions ... (and), consequently I only considered these proportions in their general aspects*. (*DM*2) How, as pure relations, do they attain applicability to objects; or alternatively, how can they be adequate to space-time events?

Once the nature of mathematics as a system of pure relations is established the answer to the rationality of its applicability can *only* be one: Because, for *scientia*,

real objects are considered only in so far as they exhibit the relations dealt with in mathematical constructs. And, *only* in this respect, they can be intended or 'meant' by those constructs. In the way a medical practitioner considers a patient only as an organism (one in which body and mind are united) presenting specific symptoms. That is why, *in order to construct the world as the subject of certain knowledge it has to be mathematized*. Once that is done, the possibility, nay, the very necessity of synthetic *a priori* judgements becomes inevitable. Such is the Cartesian procedure. It not only shows the operations of Mind through which knowledge is possible. It also shows the role played by the experimental procedure in the scientific scenario.

Briefly stated, the difference between the two systems is that Kant facing the *fact* of physics as science attempts to explain how space-time events become mathematized; whereas Descartes shows that the only way to know nature with certainty is to mathematize knowledge; moreover, Kant bases the unity of the cognitive act in the unity of the knowing subject. The Kantian analysis, however, attempts to deduce the possibility of the principles which permit scientific judgements from the Aristotelian Table of Judgements, *an impossible enterprise in the case of mathematical functions*. In contrast, the novelty, greater coherence and truthfulness of the Cartesian system originates in the fact that it acknowledges the formal certainty of mathematics. Cartesius also exhibits greater insight when, taking mathematics as a fact, he induces the nature of the cognitive Mind. Hence the existence of *Mathesis Universalis*; or its equivalent, a *Universal System of Reason*.

With the benefit of hindsight, however, it appears that the Cartesian insight into *Mathesis Universalis* was not developed fully in its logical epistemic consequences. This major speculative task became the achievement of Frege whose paramount logical and epistemological insights have attained recognition in academia only after decades of benevolent neglect.

Footnotes

(1) Quotations from *Discourse de la Méthode* are given as DM with a numeral indicating the part containing it, e.g. DM 5.

(2) The epistemic and logical relations existent between propositions, judgements, subjective psychology and objective truth were used by Frege for his reformulation of logic. Truth is not a part of a though. We can grasp a thought independently from its truth or falsity -without uttering it as a judgement. Both grasping a thought and uttering a judgement are acts of a knowing subject and belong to psychology. Nonetheless, both acts involve something that does not belong to psychology, namely the thought itself. From "Aufzeichnungen für ludwing Darmstaedter" in Gottlog Frege, *Nachgelassene Schriften*, Erster Band, Felix Meiner Verlag, Hamburg.

(3) For an exposition of the principles and concepts used during the filling-in process see Alfred Tarski's *Introduction to Logic*, specially Part II. It is our contention that Cartesius

became aware of the stages undergone in the cognitive process of applying mathematics to space-time phenomena through his invention of Analytic Geometry. See, infra, Section II.

(4) Letter of 22 février 1638 to R.P. Vatier, Professor at La Flèche.

(5) Many recent works in philosophy discuss this Fregean doctrine presented in *Über Sinn und Bedeutung*. See, e.g. William and Martha Kneale in *The Development of Logic*, Oxford, 1962; or Gunther Patzig's *Sprache und Logik*, 1980; or my own *Klassenlogik und Formale Einteilung der Wissenschaft* Inaugural-Dissertation, Free University, Berlin , 1963, Kapitel VIII.

(6) Another interesting contribution to the meaning of truth and falsity as qualities of logical-linguistic utterances originates in Bernhard Bolzano's notion of truths in themselves (*Wahrheiten an sich*).

(7) The problematic *Zeitgeist* during this transition period finds an adequate description: *The distinctive position Mersenne shared with Galileo, Gassendi and Descartes placed them then with the Aristotelians in defending the exclusive and discoverable rationality of nature against the magicians and skeptics, against the Aristotelians in proposing a different concep-tion of the kind of rational nature actually in existence, but against all other contemporary philosophies in closing many currently open questions by insisting upon specified rational criteria for admitting questions as well as answers into natural science*, in A. C. Crombie´s *Science, Optics and Music in Mediaeval and early Modern Thought*. The Hambledon Press, Page. 406.

(8) In *Leviathan*, Chap XIII, Hobbes makes a similar comment ,chap. XIII, Hobbes makes a similar comment explaining the reason for such belief: *For such is the nature of men , that however they may acknowledge many others to be more witty, or more eloquent, or more learned, yet they will hardly believe there be many as wise as themselves for they see their own wit at hand and other men's at a distance...*

(9) According to footnote 1, Chapter 1 of Bernard Williams' *Descartes, The Project of Pure Inquiry*, Huyghens, referring to the events of 10. XI. 1619 commenting Baillet's life of Descartes said, *That passage where he tells how he had his brain overheated and capable of visions...*

(10) A change in personal policy, morally motivated, similar to that of Einstein when, aban-doning pacifism, decided to cooperate with the research leading to the atomic bomb. The historical scenario undergoes significant change!

(11) To contribute to the welfare of mankind constitutes for Cartesius a moral obligation as strong as seeking peace and abandoning the state of war is for Hobbes: *The first and fundamental Law of Nature which is to seek peace, and follow it. Leviathan*, Chap. 14.

(12) Cartesian scholars tend to agree with Desmond M. Clarke's statement that the four methodological rules which are proposed in the *Discourse* provide almost no information at all, even to the sympathetic reader, about Descartes famous method. Cf. Clarke's, *Descartes philosophy of science*, Chap. 7# 22. It is the purpose of the present Essay to show that insight into the famous method is attained only when the epistemic process leading to the invention of Analytic Geometry is clearly grasped. Such a process constitutes the theme of Section II, infra.

(13) See, supra, footnote 3. There he says to R.P. Vatier, among other interesting remarks concerning DM: *Je vous dirai premièrement que mon dessein n'a point été d'en dire assez pour faire juger que les nouvelles opinions...*

(14) See above 1.2 the parallel to the Thalesian explanation regarding the moon's light and his own deduction of scientia from first principles.

(15) Such is the argument of Cassirer's *Substanz und Funktionsbegriff*, in a nutshell.

(16) Quotations from Kant follow Raymund Schmidt's edition of *Critique of Pure Reason*. The letter B followed by a number indicates the page of the 2nd Edition.

(17) Such is the explanation given at the beginning of DM6.

(18) For further details on the Kantian system based, *à la Cartesius*, on a reversal of the serial order adopted by naive realism see, M. Laserna, *Kantian Epistemology: A Copernican, or a Thalesian revolution?*, Philosophia Naturalis, 1987, Band 24, Heft 2, in which I provide abundant textual support for the argument that the entire purpose and method of the Kantian Critique, viewed under the traditional *Copernican Revolution* interpretation, constitutes a serious misreading of the Kantian text, and of its interpretation of the Thalesian demonstrative method used in geometry. Quotations from Kant follow Raymund Schmidt's edition of KdrV. The letter B followed by a number indicates the page of the 2nd Edition.

(19) Cf. Prolegomena, *Vorrede, from Ich gestehe frei: Die Errinerung des David Hume war eben dasjenige, was mir vor vielen jahren zuerts den dogmatischen Schlummer unterbrach und meinen Untersuchungen im Felde der speculativen Philosophie eine ganz andere Richtung gab ... ich versuchte also zuerts, ob sich nicht Hues Einwurf allgemein vorstellen liesse ....* The common enemy, shared by Kant and Cartesius, is skepticism. For Kant represented in the historical Hume; for Cartesianism the spirit of Humism which he himself construed as the principle of universal doubt.

(20) *Prolegomena*, 272 (Quotations from *Prolegomena* follow the pagination of the Academie-Ausgabe. The page in Arabic numerals is indicated).

(21) Quoted in Alistair C. Crombie, *Experimental science and the Rational Artist in Early Modern Europe*. Daedalus, Summer 1986.

(22) Kant, in an often overlooked foot-note in the Appendix to *Prolegomena*, and in order to avoid an intellectual intuition competing with the sensorial one, proposes that the applicability of geometrical constructs originates in the fact that *the senses are capable of also intuiting a priori.* A solution , which he says, prior to him *nobody had conceived as a possibility*. From what follows in the Lockian text it is quite clear that Locke, among others, took it for granted.

(23) Cartesius would argue that Mind *following its own rationality* imposes a notion of space-time as its own constructs; not a something given independently from the cognitive process. Such is the meaning of the statement by Mersenne, supra, footnote 19.

(24) For a more extensive discussion of the non-inductive demonstrative method employed by the scientists of the 17th Century see: Mario Laserna, *Wissenschaft als Demonstration bei Hobbes und Bacon*, Wiener Jahrbuch für Philosophie, Band XXI/ 1989.

(25) In the 'Entretein avec Burman' an identical view differentiating mathematics form physics is expounded: *C'est ainsi que toutes les démostrations des mathématicienes portent sur des êtres et sur des objets vrais et que l'objet tout entier des mathématiques, avec tout celles qu'elles y considerent est un être vrais et réel ... La différence est seulement en ceci que la physique considére son objet non seulement comme un être vrai et réel, mais comme un être en acte, et en tant que existent; les mathématiques au contraire seulement en tant que possible n'existant point en acte dans l'espace, pouvant toutefois exister.*

(26) For further discussion of the epistemic differentiation between experience and experiment see, Mario Laserna, *Knowledge, Experience and Experiment in Kant's Critique of Pure Reason*. Philosophia Naturalis, Band 19, Heft 1-2.

(27) Personal letter from Albert Einstein to Mario Laserna, of 22. IX. 1953.

(28) On the relation of mathematics to physics, see Lilianne Rivka Kfia, *The Ontological Status of Mathematical Entities*. The Review of Metaphysics, Vol. XLVII, No.1. September 1993.

(29) The logic of discovery or invention, in contrast to the logic of the syllogism, was a theme frequently discussed by the founding fathers of the Scientific Revolution. In his Two New Sciences, Second Day, Sagredo says: 'Logic, it appears to me, teaches how to do test the conclusiveness of any argument or demonstration; but I do not believe it teaches us how to discover correct arguments and demonstrations...'

(30) Cf. footnotes 26 and 27.

(31) A more ambiguous position as to the epistemic status of Mind, esprit, Geist, soul, alma, is taken by Laurence O. McKinney in his *Neurotheology, Virtual religion in the 21st Century*, 1994.

(32) Among them, for example, Antonio R. Damasio's, *Descartes' Error*, 1994.